Algebra

When $f(x)$ is divided by $(x-a)$,
the reminder will be $f(a)$

If $f(a) = 0$, then $(x-a)$ is a factor of $f(x)$

Logarithms

- $a^b = c \Rightarrow \log_a c = b$

- $\log(1) = 0$
- $\log(-ve)$ doesn't exist

- $\log(ab) = \log a + \log b$
- $\log\left(\frac{a}{b}\right) = \log a - \log b$

- $\log a^m = m \log a$

Exponential Functions

$y = a^x$; $a > 0$ is an exponential function.

$y = e^x$

$\frac{d}{dx}(e^x) = e^x$

$y = -e^x$ is the reflection of $y = e^x$ on the x-axis

$y = e^{-x}$ is the reflection of $y = e^x$ on the y-axis.

$\log_e x = \ln x$

$e^x = y \Rightarrow \ln y = x$

$\ln 1 = 0$

$\ln 0 = \infty$

$\ln e = 1$

$\ln e^2 = 2$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(\ln(x^2-1)) = \frac{2x}{x^2-1}$

$y = ax^n \Rightarrow \ln y = n \ln x + \ln a$

$\ln y$ against $\ln x$

$m = n \quad c = \ln a$

$y = ab^x \Rightarrow \ln y = x \ln b + \ln a$

$\ln y$ against $\cancel{\ln}\ x$

$m = \ln b \quad c = \ln a$

Modulus

$|x| = x \quad ; \quad x \geqslant 0$

$|x| = -x \quad ; \quad x < 0$

If $|a| = |b|$

then, $a = \pm b$

i.e $a^2 = b^2$

$\therefore |a| = |b| \iff a^2 + b^2$

Differenciation

$$\frac{d}{dx}(ab) = b \times \frac{d}{dx}(a) + a \times \frac{d}{dx}(b)$$

$a = f(x), \quad b = g(x)$

$$\frac{d}{dx}\left(\frac{a}{b}\right) = \frac{b \times \frac{d}{dx}(a) - a \times \frac{d}{dx}(b)}{b^2}$$

Trignometry

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A-B) = \dfrac{\tan A + \tan B}{1 + \tan A \tan B}$

- $\operatorname{cosec} \theta = \dfrac{1}{\sin \theta}$
- $\sec \theta = \dfrac{1}{\cos \theta}$
- $\cot \theta = \dfrac{1}{\tan \theta} = \dfrac{\cos \theta}{\sin \theta}$
- $\sin^2 \theta + \cos^2 \theta = 1$

  $\sin^2 \theta = 1 - \cos^2 \theta$

  $\cos^2 \theta = 1 - \sin^2 \theta$
- $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$
- $\sec^2 \theta - \tan^2 \theta = 1$
- $\sin 2\theta = 2 \sin \theta \cos \theta$
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
- $\cos 2\theta = 1 - 2\sin^2 \theta$
- $\cos 2\theta = 2\cos^2 \theta - 1$
- $\tan 2\theta = \dfrac{2 \tan \theta}{1 - \tan^2 \theta}$

- $a \cos \theta \pm b \sin \theta \Rightarrow r \cos(\theta \pm \alpha)$

  $r \sin(\theta \pm \alpha)$

  $r^2 = a^2 + b^2$
- $\tan \alpha = \dfrac{a}{b}$ or $\dfrac{b}{a}$

Differenciation (Cont.)

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}\left[\sin f(x)\right] = \cos f(x) \times f'x$
- $\frac{d}{dx}\left[\cos(ax+b)\right] = -a\sin(ax+b)$
- $\frac{d}{dx}(\sin^n x) = n\sin^{n-1} x \times \cos x$
- $\frac{d}{dx}(\cos^n x) = n\cos^{n-1} x \times -\sin x$

Integration

$$\int (ax+b)^n \, dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + k$$

$$\int e^x \, dx = e^x + k$$

$$\int ce^x \, dx = ce^x + k$$

$$\int e^{(ax+b)} \, dx = \frac{1}{a} e^{(ax+b)} + k$$

$$\int \frac{1}{x} \, dx = \ln |x| + k$$

$$\int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln |ax+b| + k$$

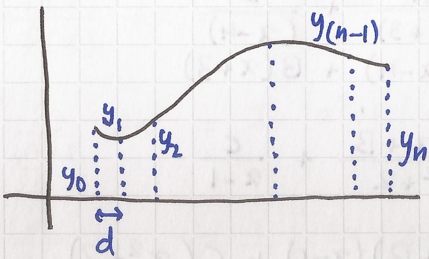
$$\int \sin x \, dx = -\cos x + k$$

$$\int \cos x \, dx = \sin x + k$$

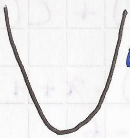
$$\int \sec^2 x \, dx = \tan x + k$$

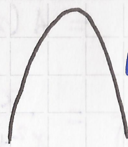
$$\int \sin(ax+b) \, dx = \frac{-1}{a} \cos(ax+b) + k$$

Trapezium Rule



$$A \approx \frac{1}{2} d \left(y_0 + y_n + 2y_1 + 2y_2 + \dots \, 2y(n-1) \right)$$

 overestimate

 underestimate

Numerical Solutions of Equations

In $f(x)=0$, if $f(x_1)$ and $f(x_2)$ are opposite in sign and if the curve is continuous in the interval (x_1, x_2) then there is a root in the given interval.

Partial fractions

- $\dfrac{x-2}{(x+3)(x-4)} = \dfrac{A}{(x+3)} + \dfrac{B}{(x-4)}$

$$x-2 = A(x-4) + B(x+3)$$

- $\dfrac{x^2+1}{(x^2+2)(x-1)} = \dfrac{Ax+B}{x^2+2} + \dfrac{C}{x-1}$

$$x^2+1 = (Ax+B)(x-1) + C(x^2+2)$$

- $\dfrac{x-1}{(x+1)(x+2)^2} = \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$

$$x-1 = A(x+2)^2 + B(x+1)(x+2) + C(x+1)$$

- $\dfrac{x^2-2}{(x+3)(x-2)} = \dfrac{x^2-2}{x^2+x+6} = \dfrac{(x^2+x-6)-x+4}{x^2+x-6}$

$$= 1 + \frac{-x+4}{x^2+x-6}$$

Integration (cont.)

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + k$$

$$\int \frac{2x}{x^2+2}\,dx = \ln|x^2+2| + k$$

$$\int \frac{x}{x^2+2}\,dx = \frac{1}{2}\int \frac{2x}{x^2+2}\,dx$$
$$= \frac{1}{2}\ln|x^2+2| + k$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$
$$= -\ln|\cos x| + k$$

Integration by parts - Product Rule

$$\int \underbrace{f(x)}_{1^{st}}\,\underbrace{g(x)}_{2^{nd}}\,dx = \underbrace{f(x)}_{1^{st}}\underbrace{\int g(x)\,dx}_{I\ 2^{nd}} - \underbrace{\frac{d}{dx}f(x)}_{D1^{st}}\underbrace{\int g(x)\,dx}_{I\ 2nd.}$$

$f(x) = \text{LATE}$

L - log
A - algebra
T - trig
E - e